

Homework 2

(Due date: February 3rd @ 5:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (38 PTS)

- a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher bit. (8 pts)

Example ($n=8$):

✓ $54 + 210$

$$\begin{array}{r} \overset{c_7}{1} \overset{c_6}{1} \overset{c_5}{1} \overset{c_4}{1} \overset{c_3}{1} \overset{c_2}{1} \overset{c_1}{1} \overset{c_0}{1} \\ 54 = 0x36 = 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 + \\ 210 = 0xD2 = 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \\ \hline \text{Overflow!} \rightarrow 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \end{array}$$

✓ $77 - 194$

$$\begin{array}{r} \text{Borrow out!} \rightarrow b_8=1 \\ 77 = 0x4D = 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 - \\ 194 = 0xC2 = 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \\ \hline 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \end{array}$$

- ✓ $23 + 403$
- ✓ $103 + 204$

- ✓ $77 - 128$
- ✓ $199 - 107$

- b) We need to perform the following operations, where numbers are represented in 2's complement: (24 pts)

- ✓ $-61 + 128$
- ✓ $225 + 31$
- ✓ $256 - 257$

- ✓ $-126 + 263$
- ✓ $-511 - 167$
- ✓ $137 + 886$

- For each case:

- ✓ Determine the minimum number of bits required to represent both summands. You might need to sign-extend one of the summands, since for proper summation, both summands must have the same number of bits.
- ✓ Perform the binary addition in 2's complement arithmetic. The result must have the same number of bits as the summands.
- ✓ Determine whether there is overflow by:
 - i. Using c_n, c_{n-1} (carries).
 - ii. Performing the operation in the decimal system and checking whether the result is within the allowed range for n bits, where n is the minimum number of bits for the summands.
- ✓ If we want to avoid overflow, what is the minimum number of bits required to represent both the summands and the result?

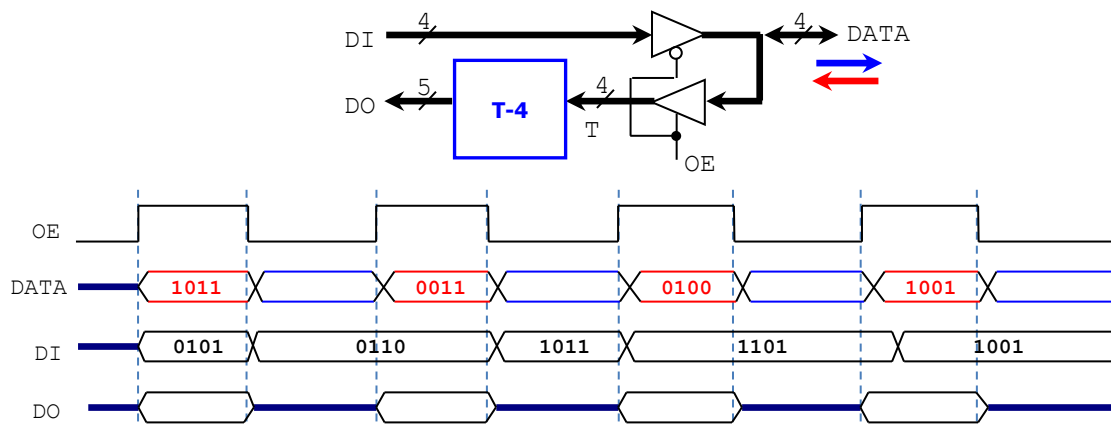
- c) Perform the multiplication of the following numbers that are represented in 2's complement arithmetic with 4 bits. (6 pts)

✓ 0101×0111 , 0101×1001 , 1100×1010

PROBLEM 2 (7 PTS)

- Complete the timing diagram (signals DO and $DATA$) of the following circuit. The circuit in the blue box computes the signed operation T-4, with the result having 5 bits. T is a 4-bit signed (2C) number.

For example: if $T=1010 \rightarrow DO = 1010 - 0100 = 11010 + 11100 = 10110$.



PROBLEM 3 (29 PTS)

- In these problems, you MUST show your conversion procedure. **No procedure = zero points.**
- a) Convert the following decimal numbers to their 2's complement representations: binary and hexadecimal. (9 pts.)
 - ✓ -255.6875, 31.625, -128.6875
- b) Complete the following table. The decimal numbers are unsigned: (6 pts.)

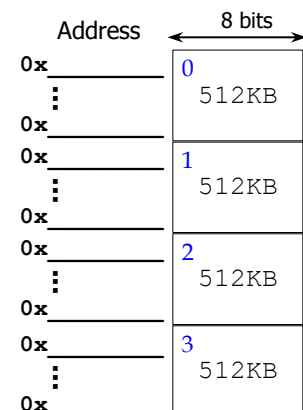
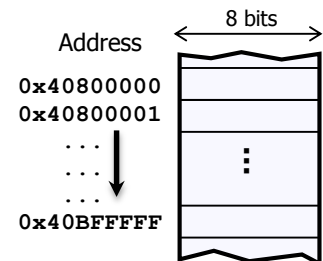
| Decimal | BCD | Binary | Reflective Gray Code |
|---------|--------------|----------|----------------------|
| 127 | | | |
| | | 10111010 | |
| 512 | | | |
| | | | 10010101 |
| | | 11101010 | |
| | 100001110101 | | |

- c) Complete the following table. Use the fewest number of bits in each case: (14 pts.)

| REPRESENTATION | | | |
|----------------|--------------------|----------------|----------------|
| Decimal | Sign-and-magnitude | 1's complement | 2's complement |
| -120 | | | |
| | | | 10101000 |
| | | 0111010001 | |
| | | | 1000000 |
| | 1001111 | | |
| -64 | | | |
| | | | 10000011 |

PROBLEM 4 (26 PTS)

- a) What is the minimum number of bits required to represent: (2 pts)
 - ✓ 32678 memory addresses in a computer?
 - ✓ Numbers between 0 and 2048?
- b) A microprocessor has a 32-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (6 pts)
 - What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? $1\text{KB} = 2^{10}$ bytes, $1\text{MB} = 2^{20}$ bytes, $1\text{GB} = 2^{30}$ bytes
 - A memory device is connected to the microprocessor. Based on the size of the memory, the microprocessor has assigned the addresses $0\text{x}40800000$ to $0\text{x}40\text{BFFFFFF}$ to this memory device.
 - What is the size (in bytes, KB, or MB) of this memory device?
 - What is the minimum number of bits required to represent the addresses only for this memory device?
- c) A microprocessor has a memory space of 2 MB. The size of the memory contents of each address is 8 bits (1 byte). (7 pts)
 - ✓ What is the address bus size (number of bits of the address) of this microprocessor?
 - ✓ What is the range (lowest to highest, in hexadecimal) of the memory space for this microprocessor?
 - ✓ The figure (right) shows four memory chips that are placed in the given positions:
 - Complete the address ranges (lowest to highest, in hexadecimal) for each of the memory chips. (5 pts)



- d) The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. (11 pts)
- What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor?
 - If we have a memory chip of 8MB, how many bits do we require to address 8MB of memory? (1 pt.)
 - We want to connect the 8MB memory chip to the microprocessor. For optimal implementation, we must place those 8MB in an address range where every single address share some MSBs (e.g.: $0x0000000$ to $0x07FFFFFF$). Provide a list of all the possible address ranges that the 8MB memory chip can occupy. You can only use any of the non-occupied portions of the memory space as shown below. (8 pts)

